IR-1553

M. A. / M. Sc. (First Semester) Examination, Dec. 2021

MATHEMATICS

Paper : Third

(Topology-I)

Time Allowed : Three hours

Maximum Marks : 40

Note : *Attempt questions of all **two** sections as directed. Distribution of marks is given with sections.*

Section-A

(Short Answer Type Questions) 5×3=15

Note : *Attempt all the questions. Each question carries 3 marks.*

Unit-I

1. Define a countable set. Show that a subset of a countable set is countable.

Or

Write the statement of Cantor's theorem and the continuum hypothesis.

Unit-II

2. Define the closure of a set in a topological space $(X, \mathcal{J})$. Show that for $A, B \subset X$,

$$\overline{A \cup B} = \overline{A} \cup \overline{B}$$

Or

Define the interior of a set in a topological space $(X, \mathcal{J})$. Show that for $A, B \subset X$,

$$\operatorname{int}(A \cap B) = \operatorname{int} A \cap \operatorname{int} B$$

Unit-III

3. Define base for a topological space $(X, \mathcal{J})$ and give one examples.

Or

Write the definition of a continuous map $f : X \to Y$. Show that a constant map $C : X \to Y$ is continuous.

Unit-IV

4. Define a first countable space and give one example.

Or

Define a separable space and give one example.

Unit-V

5. Define a component in a topological space $(X, \mathcal{J})$. Show that components are closed sets in X.

Or

Give example of a connected space which is not locally connected. Also give one example of a locally connected space which is not connected.

Section-B

(Long Answer Type Questions) 5×5=25

Note : Attempt all the five questions. Each question carries 5 marks.

Unit-I

6. Show that a countable union of countable sets is countable.

Or

Show that the following statements for a set A are equivalent :

(i) There exists an injective function $f : Z_{+} \to A$.

(ii) There exists a bijection of A with a proper subset of itself.

(iii) A is infinite.

Unit-II

7. Let $X \neq \phi$ be a set and $\{\mathfrak{J}_\alpha \mid \alpha \in \mathfrak{J}\}$ be a family of topologies on X. Show that $\mathfrak{J} = \bigcap_{\alpha \in \mathfrak{J}} \mathfrak{J}_\alpha$ is again a topology on X.

Or

By using the de Morgan's rule show that in a topological space $(X, \mathfrak{J})$, an arbitrary intersection of closed sets is closed and a finite union of closed sets is closed.

Unit-III

8. Let X be a nonempty set and $\mathfrak{J}, \mathfrak{J}'$ be two topologies on X having bases $\mathcal{B}$ and $\mathcal{B}'$ respectively. Show that $\mathfrak{J}'$ is finer than $\mathfrak{J}$ iff for each $x \in X$ and each basis element $B \in \mathcal{B}$ containing x, there is a basis element $B' \in \mathcal{B}'$ such that $x \in B' \subset B$.

Or

Define a continuous map between two topological spaces $(X, \mathfrak{J})$ and $(Y, \mathfrak{J}')$ and give one example. Show that the composition of two continuous maps is again continuous.

Unit-IV

9. Show that a second countable is first countable. Give one example of a first countable space which is not second countables.

Unit-V

10. Define a path connected space. Show that a path connected space is connected. Give one example of a connected space which is not path connected.

Or

Show that a topological space (X, J) is locally connected if and only if for every open set $\cup$ of X, each component of $\cup$ is open in X.