JR-3531

M. A. / M. Sc. (Second Semester) Examination, June 2022

MATHEMATICS

Paper : Fifth (i) (Optional)

(Differential Equation-II)

Time Allowed : Three hours

Maximum Marks : 40

Note : *Attempt questions of all* ***two*** *sections as directed. Distribution of marks is given with sections.*

Section-'A'

(Short Answer Type Questions) **5×3=15**

Note : *Attempt all* ***five*** *questions. Each question carries 03 marks.*

Unit-I

1. Explain continuity with one example.

Or

Explain higher order differentiability with one example.

Unit-II

2. Explain Autonomous system.

Or

Define rotation points with one example.

Unit-III

3. Describe Nonoscillatory equations and principal solution.

Or

State Nonoscillation theorem.

Unit-IV

4. Explain the use of implicit function.

Or

Describe non-linear problem.

Unit-V

5. Explain second order boundary value problems.

Or

Explain linear problem in second order boundary value problem.

Section-'B'

(Long Answer Type Questions) 5×5=25

Note : *Answer all five questions. Each question carries 05 marks.*

Unit-I

6. Let $f(t, y, z)$ be continuous on an open (t, y, z) -- set E with property that for every $(t_0, y_0, z) \in E$, the initial value property.

$$y' = f(t, y, z) \text{ and } y(t_0) = y_0$$

where for each fixed z has a unique solution $y(t) \equiv \eta(t, t_0, y_0, z)$. Let $w_- < t < w_+$ be the maximum interval of existence of $y(t) = \eta(t, t_0, y_0\ z)$. Then

$w_+ = w_+(t_0, y, z)$ in a lower (or upper) semi-continuous fucntion of $(t_0, y_0, z) \in E$ and η (t, t_0, y_0, z) is continuous on the set $w_- < t < w_+$, $(t_0, y_0, z) \in E$.

Or

Let $f(t, y, z, z^*)$ be a continuous function on an open (t, y, z, z^*). Set E, where z^* is a vector of any dimention. Suppose that f has continuous first order partial derivative with respect to the components of y and z. Then

$$y' = f(t, y, z, z^*) \quad y(t_0) = y_0$$

have a unique solution $\eta = (t, t_0, y_0, z, z^*)$ for fixed z, z^* with $(t_0, y_0, z, z^*) \in E$ η has first order partials with respect to t, t_0 the components of y and of z, and the second order partial

$$\frac{\partial^2 \eta}{\partial t \partial t_0}, \frac{\partial^2 \eta}{\partial t \partial y_0^k}, \frac{\partial^2 \eta}{\partial t \partial z'}$$

Finally there partial of η are continuous with respect to (t, t_0, y_0, z, z^*).

Unit-II

7. Let $f(y) = f(y^1, y^2)$ be continuous on an open plane set E and Let C^+; $y = y_+(t)$ be a solution of

$$y' = f(y)$$

for $t \geq 0$ with a compact closure in E. In addition suppose that

$$y_+(t_1) \neq y_+(t_2)$$

for $0 \leq t_1 \leq t_2 < \infty$ and the $\Omega(C^+)$ contain no stationary points. Then $\Omega(C^+)$ in the set of points y on a periodic solution C_p: $y = y_p(t)$ of $y' = f(y)$.

Or

Explain Foci, Nodes and Saddle points.

PTO

Unit-III

8. State and prove that storm's first comparison theorem.

Or

Let $f_1, f_2,$ .... be a sequence of elements of $L^2(a,b)$ satisfying $\| f_n \| \leq 1$. Then three exists an $f(t) \in L^2(a,b)$ and a subsequence $f_n(1)^{(t)}\ f_n(2)^{(t)},$ .... of the given sequence such that $\| f \| \leq 1$ and $f_n(j) \to f(t)$ weakly as $i \to \infty$.

Unit-IV

9. Let $A(t)$ be continuous for $0 \leq t \leq p$ M, N constants $d \times d$ matrices such that the $d \times 2d$ matrix (M, N) is of rank d. Then

$$y' = A(t)y + g(t)$$

has a solution $y(t)$ satisfying

$$M\ y(0) - N\ y(p) = 0$$

for every continuous $g(t)$ if and only if $y' = A(t)y$ and

$$M\ y(a) - N\ y(p) = 0$$

has no non-trivial solution in which can $y(p)$ is unique and there exists a constant K independent of $g(t)$ such that

$$\| y(t) \| \leq k \int_0^p \| g(s) \| \, ds \text{ for } 0 \leq t \leq p.$$

Or

Explain linear equation with example.

Unit-V

10. Let $B(t)$, $F(t)$ be continuous for $0 \leq t \leq p$. Then

$$x'' = B(t)x + F(t)x' + h(t)$$

has a solution $x(t)$ satisfying

$$x(0) = 0,\ x(p) = 0$$