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JR-3524

M. A./M. Sc. (Second Semester) Examination, June 2022

MATHEMATICS

Paper : Fourth

(Complex Analysis-II)

Time Allowed : Three hours

Maximum Marks : 40

Note : *Attempt questions of all **two** sections as directed. Distribution of marks is given with sections.*

Section-'A'

(Short Answer Type Questions) **5×3=15**

Note : *Attempt all **five** questions. Each question carries 3 marks.*

1. Define Riemann zeta function.

Or

Define Gamma function.

2. Prove that there can not be more than one continuation of an analytic function $f(z)$ into the same domain.

Or

Define natural boundary.

3. Define harmonic functions on a dist with an example.

Or

State only mean value theorem.

4. Define Green's function.

Or

Define order of an entire function.

5. Define the range of an analytic function.

Or

State only montel caratheodary theorem.

Section-'B'

(Long Answer Type Questions) **5×5=25**

Note : *Attempt all five questions. Each question carries 5 marks.*

6. State and prove Weierstrass factorization theorem.

Or

Show that for any $z \in c, \frac{1}{2}|z| \leq |\log(1+z)| \leq \frac{3}{2}z$.

7. State and prove Runge's theorem.

Or

Let σ be the contour constructed curve. Let f be an analytic function in G, then

$$f(z_0) = \frac{1}{2\pi i}\int_{\sigma} \frac{f(z)}{(z - z_0)} dz$$

8. State and prove Schwarz Reflection principle.

Or

State and prove monodromy theorem.

 PTO

9. State and prove Hodamad's factorization theorem.

Or

Write a short note on :

(i) Exponent of convergence

(ii) Dirichlet problem

10. Let $f \in s$ with power series given by

$$f(z) = \frac{z}{\left(1 + z\, e^{i\theta}\right)^2}; 0 \leq \theta \leq 2\pi$$

Then prove that $|a_z| \leq 2$, equility occurs of and only of f is a rotation of the Koebe function.

Or

State and prove 1/4 theorem.