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IR-1561

M. A. / M. Sc. (Third Semester) Examination, Dec. 2021

MATHEMATICS

(Compulsory)

Paper : First

(Integration Theory and Functional Analysis-I)

Time Allowed : Three hours

Maximum Marks : 40

Note : *Attempt questions of all **two** sections as directed. Distribution of marks is given with sections.*

Section–'A'

(Short Answer Type Questions) 5×3=15

Note : *Attempt all the **five** questions. **One** question from each unit is compulsory. Each question carries 3 marks.*

Unit-I

1. Explain signed measure in details.

Or

State and prove Hahn decomposition theorem.

Unit-II

2. Define outer measure and measurability.

Or

Prove that if $A \in a$ and if $\langle A_i \rangle$ is any sequence of set in a such that :

$$A \subset \bigcup_{i=1}^{\infty} A_i \text{ then } \mu A \Sigma \sum_{i=1}^{\infty} \mu A_i .$$

Unit-III

3. Define normed space, Banach space and Quotient space.

Or

A subspace Y of a Banach space X is complete if and only if the set Y is closed in X.

Unit-IV

4. Let T be a bounded linear operator, then show that an alternative formula for the norm of T is :

$$\| T \| = \sup_{\substack{x \in D(T) \\ \| x \| = 1}} \| T_x \|$$

Or

Define identity operator, integration operator and linear operator in details.

Unit-V

5. Define second algebraic dual space and define linear functional and bounded linear functional.

Or

Describe canonical mapping.

Section–'B'

(Long Answer Type Questions) 5×5=25

Note : *Attempt all five questions.* ***One*** *question from each unit is compulsory. Each question carries 5 marks.*

PTO

Unit-I

6. Let (χ, β, μ) be a finite measure space and g and integrable function such that for same constant M,

$$\left| \int_g \phi \, d\mu \right| \leq M \| \phi \|_p$$

for all simple function ϕ. Then $g \in L^q$.

Or

State and prove Lebesque decomposition theorem.

Unit-II

7. Let μ be a measure an algebra a, μ^* be an outer measure induced by μ and E be any set. Then for $\in > 0$ there is a set. $A \in a_\sigma$ with $E \subset A$ and $\mu^* A \sum \mu^* E + \in$, there is *abo* a set $B \in a_{\sigma\delta}$ with $E \subset B$ and $\mu^* E = \mu^* \beta$.

Or

Show that the class $\mathcal{B}$ of μ^*-measurable sets is a σ-algebra. If $\bar{\mu}$ is μ^* restricted to $\mathcal{B}$, then $\bar{\mu}$ is a complete on $\mathcal{B}$.

Unit-III

8. Every finite dimensional subspace Y of a normed space X is complete.

Or

Let Y and Z be a subspaces of a normed space X, and suppose that Y is a closed and is a proper subset of Z. Then for every real number θ in the internal $(0, 1)$, there is a $z \in Z$ such that

$$\| z \| = 1, \; \| z - y \| \geq \theta \;\; \forall \; y \in Y$$

Unit-IV

9. Let X, Y be vector spaces, both real or both complex. Let $T : \mathcal{D}(T) \to Y$ be linear operator with domain $\mathcal{D}(T) \subset X$ and range $\mathcal{R}(T) \subset Y$. Then

(a) The inverse $T^{-1} : R(T) \to \mathcal{D}(T)$ exists if and only if $T_x = 0 \Rightarrow x = 0$

(b) If T^{-1} exists, it is a linear operator.

(c) If $\dim \mathcal{D}(T) = n < \infty$ and T^{-1} exists, then $\dim R(T) = \dim \mathcal{D}(T)$

PTO

Or

Let $T : \mathcal{D}(T) \to Y$ be a linear operator, where $\mathcal{D}(T) \subset X$ and X and Y are normed spaces then :

(a) T is continuous if and only if T is bounded.

(b) If T is continuous at a single point it is continuous.

Unit-V

10. Let X be a an n-dimensional vector space and

$E = \{e_1, \ldots\ldots\ldots, e_n\}$ as basis for X. Then

$F = \{ f_1, \ldots\ldots\ldots, f_n \}$ given by :

$$f_k(e_j) = \delta_{jk} = \begin{cases} 0 & \text{if} \quad j \neq k \\ 1 & \text{if} \quad j = k \end{cases}$$

is a basis for the algebric dual X^* of X and $\dim X^* = \dim X = n$.

Or

If Y is a Banach space. Then $\mathcal{B}(X, Y)$ is a Banach space.