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IR-1566

M. A. / M. Sc. (Third Semester) Examination, Dec. 2021

MATHEMATICS

(Group-III) (I-Optional)

Paper : First

(Theory of Linear Operators-I)

Time Allowed : Three hours

Maximum Marks : 40

Note : *Attempt all questions. All symbols have their usual meaning.*

Section-A

(Short Answer Type Questions) $5 \times 3 = 15$

Note : *Attempt all the* ***five*** *questions.* ***One*** *question from each unit is compulsory. Each question carries 03 marks.*

Unit-I

1. Write the definition of the following :

   (a) Eigen values

   (b) Eigen function

   (c) Spectrum

Or

State only spectral mapping theorem.

Unit-II

2. Define Algebra, Normed algebra and Banach algebra.

Or

Define spectrum and spectrum radius.

Unit-III

3. Let $T : X \to X$ be a compact linear operator and $S : X \to X$ a bounded linear operator on a normed space X. Then TS and ST are compact.

Or

Explain solvability of operators equations.

Unit-IV

4. Define Complex Hilbert space, Adjoint operator and Bounded self adjoint operator.

Or

Explain Resolvent set and Positive operators in details.

Unit-V

5. Let P_1 and P_2 be projection on a Hilbert space H. Show that their sum $P_1 + P_2$ is a projection on H, if and only if $Y_1 = P_1 (H)$ and $Y_2 = P_2 (H)$ are orthogonal.

Or

Explain spectral family in details.

Section-'B'

(Long Answer Type Questions) 5×5=25

Note : Attempt all the five questions. One question from each unit is compulsory. Each question carries 5 marks.

Unit-I

6. Let $T \in B(X, Y)$, where X is a Banach space. If $\|T\| < 1$, then $(I - T)^{-1}$ exists as bounded linear operator on the whole space X and

$$(I-T)^{-1} = \sum_{j=0}^{\infty} T^{j} = I + T + T^{2} + ...$$

Or

All matrices representing a given linear operator $T : X \to X$ on a finite dimensional normed space X, relative to various based for X have the same eigen values.

Unit-II

7. If $X \neq \{0\}$ is a complex Banach space and $T \in B(X, X)$, then $\sigma(T) \neq \Phi$.

Or

Let $T : X \to X$ be a compact linear operator on a normed space X. Then for every $\lambda \neq 0$ the range of $T_{\lambda} = T - \lambda I$ is closed.

Unit-III

8. Let $T : X \to X$ be a compact linear operator on a normed space X and let $\lambda \neq 0$. Then equation $Tx - \lambda x = 0$ and $T^{*} f - \lambda T = 0$ have the same number of linearly independent solutions.

Or

State and prove Fredholm Alternative for integral equation.

Unit-IV

9. For any bounded self-adjoint linear operator T on a complex Hilbert space H, we have

$$\| T \| = \max\left(|m|, |M| \right) = \sup_{\|x\|=1} \left| \langle Tx, x \rangle \right|$$

Or

If two bounded self adjoint linear operators S and T on a Hilber space H are positive and ($ST = TS$), then their product ST is positive.

Unit-V

10. Every positive bounded self adjoint linear operator $T : H \rightarrow H$ on a complex Hilbert space H has a positive square root A, which is unique. This operator A commutes with every bounded linear operator on H which commutes with T.

Or

A bounded linear operator $P : H \rightarrow H$ on a Hilbert space H is a projection if and only if P is a self-adjoint and idempotent (that is $P^2 = P$).