KR-2632

M. A. / M. Sc. (First Semester) Examination, Dec. 2022

MATHEMATICS

Paper : Second

(Real Analysis)

Time Allowed : Three hours

Maximum Marks : 40

*Note : Attempt questions of all **two** sections as directed.*

Section-'A'

(Short Answer Type Questions) **5×3=15**

*Note : Attempt all the five questions. **One** question from each unit is compulsory. Internal choice provided. Each question carries 03 marks.*

Unit-I

1. Show that :

$$\int_{-a}^{b} f\, d\alpha \leq \int_{a}^{-b} f\, d\alpha$$

Or

If $f \in \mathcal{R}(\alpha)$ on $[\,a, b\,]$ and if $|\,f(x)\,| \leq M$ on $[\,a, b\,]$ then show that :

$$\left| \int_{a}^{b} f\, d\alpha \right| \leq M\left[\alpha(b) - \alpha(a)\right]$$

Unit-II

2. Define pointwise and uniform convergence.

Or

Define the rearrangement of terms of a series.

Unit-III

3. State and prove the Weierstrass M-test.

Or

Let $f_n(x) = \dfrac{nx}{1+n^2x^2}$, when $0 \le x \le 1$ and $n = 1, 2, 3, \ldots$. Examine as to whether the sequence $\{ f_n \}$ is uniformly convergent in $[0, 1]$ or not.

Unit-IV

4. Define partial derivative.

Or

Let Ω be the set of all invertible linear operators on R^n and if $A \in \Omega$, $B \in L(R^n)$ and $\| B - A \| \cdot \| A^{-1} \| < 1$ then show that $B \in \Omega$.

Unit-V

5. If $u_1 = \dfrac{x_2 x_3}{x_1}$, $u_2 = \dfrac{x_1 x_3}{x_2}$, $u_3 = \dfrac{x_1 x_2}{x_3}$ then show that $J(u_1, u_2, u_3) = 4$.

Or

Define the differential forms.

Section-'B'

(Long Answer Type Questions) **5×5=25**

Note : *Attempt all the five questions.* ***One*** *question from each unit is compulsory. Each question carries 05 marks.*

Unit-I

6. $f \in \mathcal{R}(\alpha)$ on $[a, b]$ if and only if for every $\in > 0$ there exists a partition P such that

$$U(P, f, \alpha) - L(P, f, \alpha) < \in$$

Or

If $f \in \mathcal{R}(\alpha)$ on $[a, b]$ and if $a < c < b$ then $f \in \mathcal{R}(\alpha)$ on $[a, c]$ and on $[c, b]$ and

$$\int_a^c f\, d\alpha + \int_c^b f\, d\alpha = \int_a^b f\, d\alpha$$

Unit-II

7. If ϑ' is continuous on $[a, b]$, then ϑ is rectifiable, and

$$\wedge(\vartheta) = \int_a^b |\vartheta'(t)|\, dt$$

Or

If $\vec{f}$ maps $[a,b]$ into R^K and if $\vec{f} \in \mathcal{R}(\alpha)$ for some monotonically increasing function α on $[a,b]$, then $|\vec{f}| \in \mathcal{R}(\alpha)$ and $\left|\int_a^b \vec{f}\, d\alpha\right| \le \int_a^b |\vec{f}|\, d\alpha$.

Unit-III

8. Let α be monotonically increasing on $[a,b]$. Suppose $f_n \in \mathcal{R}(\alpha)$ on $[a,b]$, for $n = 1, 2, 3, \ldots\ldots\ldots$, and suppose $f_n \to f$ uniformly on $[a,b]$. Then $f_n \in \mathcal{R}(\alpha)$ on $[a,b]$ and

$$\int_a^b f\, d\alpha = \lim_{n\to\infty} \int_a^b f_n\, d\alpha$$

Or

The sequence of functions $\{f_n\}$ defined on E, converges uniformly on E if and only if for every $\in > 0$ there exists an integer N such that $m \ge N$, $n \ge N$, $x \in E$ implies $|f_n(x) - f_m(x)| \le \in$.

Unit-IV

9. State and prove the chain rule.

Or

State and prove the Taylor's theorem.

Unit-V

10. State and prove the Stoke's theorem.

Or

State and prove the Implicit function theorem.