LR-2152

M. A. / M. Sc. (Fourth Semester) Examination, May-June 2023

MATHEMATICS

Paper : First-(Optional Group-I)

(Advanced Functional Analysis-II)

Time Allowed : Three hours

Maximum Marks : 40

Note : *Attempt questions of all **two** sections as directed. Distribution of marks is given with sections. Symbols have their usual meanings.*

Section-'A'

(Short Answer Type Questions) **5×3=15**

Note : *Attempt all **five** questions. **One** question from each unit is compulsory. Each question carries 3 marks.*

Unit-I

1. Define Finite-dimensional spaces with example.

Or

If X is a locally bounded topological vector space with the Heine-Borel property, then X has finite dimension.

Unit-II

2. Define F-spaces with example.

Or

Write the statement of the open mapping theorem.

Unit-III

3. Define Bornological spaces with example.

Or

Define Baralled spaces with example.

Unit-IV

4. What do you mean by Schwarz spaces? Explain.

Or

Define Reflexive topological vector spaces.

Unit-V

5. Define test function spaces with example.

Or

Explain Lebesgue measurable.

Section-'B'

(Long Answer Type Questions) 5×5=25

Note : *Attempt all **five** questions. **One** question from each unit is compulsory. Each question carries 5 marks.*

Unit-I

6. State and prove Hahn-Banach theorem.

Or

If X is a locally bounded topological vector space with the Heine-Borel property then X has finite dimension.

Unit-II

7. If $B : X \times Y \to Z$ is bilinear and separately continuous, X is an F-space, and Y and Z are topological vector spaces. Then

$$B(x_n, y_n) \to B(x_0, y_0) \text{ in } z$$

whenever $x_n \to x_0$ in X and $y_n \to y_0$ in y. If X is metrizable, it follows that B is continuous.

Or

State and prove the closed graph theorem.

Unit-III

8. State and prove Bipolar theorem.

Or

Explain duality polar with example.

Unit-IV

9. Define the following :

(i) Semi-reflexive topological vector spaces

(ii) Maceckey spaces

(iii) Mantel spaces

Or

Explain Quasi-completeness.

Unit-V

10. Show that $D(\dot{\Omega})$ has the Heine-Borel property.

Or

If $\wedge$ is a linear mapping of $D(\Omega)$ into a locally convex space Y. Then :

(i) $\wedge$ is continuous

(ii) $\wedge$ is bounded